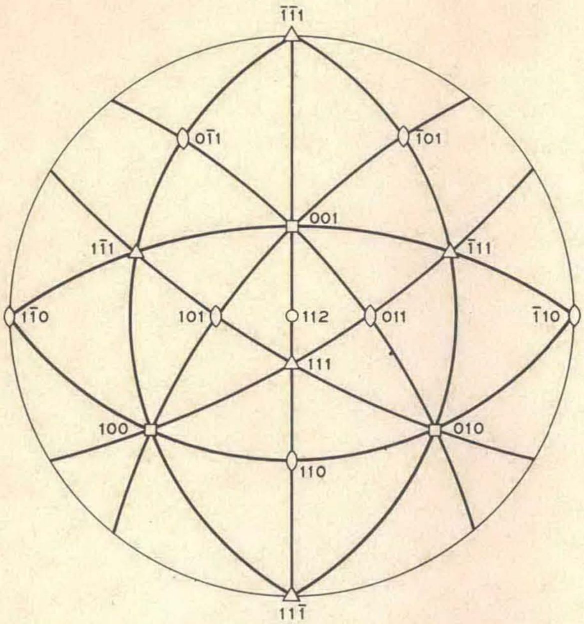

FIG. 4. Standard (112) stereographic projection of cubic crystal.

Eq. (28) means that the [001] is a hard direction for L.F. deformation, so that the easy direction on the (110) plane is along the [1̄10] rolling direction. There is no anisotropy from S.C. deformation, Eq. (29).

(c) (111) [1̄1̄2] Rolling

The specimen coordinate system specifies $x'-[111]$, $y'-[\bar{1}10]$, and $z'-[\bar{1}\bar{1}2]$. The transformation matrix is

$$\begin{array}{c|ccc} & x' & y' & z' \\ \hline X & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{6}} \\ Y & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{6}} \\ Z & \frac{1}{\sqrt{3}} & 0 & \frac{2}{\sqrt{6}}. \end{array}$$

Hence

$$\epsilon_{xx}=-r/6,\quad \epsilon_{yy}=-r/6,\quad \epsilon_{zz}=r/3,$$
$$\epsilon_{yz}=\epsilon_{zx}=-2r/3,\quad \epsilon_{xy}=-r/6. \tag{30}$$

From Fig. 2, the active slip systems based on the Tucker criterion are (4), (5), (7), and (12). Then

$$\begin{aligned} 2\epsilon_{xx}&=S_4+S_7-S_{12},\\ 2\epsilon_{yy}&=S_5-S_7+S_{12},\\ 2\epsilon_{zz}&=-S_4-S_5,\\ 4\epsilon_{yz}&=S_4+S_7+S_{12},\\ 4\epsilon_{zx}&=S_5+S_7+S_{12},\\ 4\epsilon_{xy}&=S_4+S_5. \end{aligned} \tag{31}$$

Solution of (30) and (31) gives

$$S_4=S_5=-(r/3),\quad S_7=S_{12}=-(7/6)r. \tag{32}$$

Hence

$$E_{\mathrm{LF}}=(1/48)K_{\mathrm{LF}}r[(\alpha_3^2-\alpha_2\alpha_3-\alpha_3\alpha_1)+7(\alpha_1-\alpha_2)^2]. \tag{33}$$

The second term inside the brackets dominates and places the hard direction at [1̄10], which is transverse to the [1̄1̄2] rolling direction. Similarly,

$$E_{\mathrm{SC}}=-(1/144)K_{\mathrm{SC}}r(2\alpha_2\alpha_3+2\alpha_3\alpha_1+5\alpha_1\alpha_2). \tag{34}$$

Calculation based on Eq. (34) shows that the hard direction on the rolling plane is again [1̄10].

(d) (112) [1̄10] Rolling

The specimen coordinate axes are: $x'-[112]$, $y'-[11\bar{1}]$, and $z'-[\bar{1}10]$, Fig. 4. The transformation matrix is

$$\begin{array}{c|ccc} & x' & y' & z' \\ \hline X & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ Y & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ Z & \frac{2}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & 0 \end{array}.$$

Hence

$$\epsilon_{xx}=r/3,\quad \epsilon_{yy}r/3,\quad \epsilon_{zz}=-2r/3,$$
$$\epsilon_{yz}=-r/3,\quad \epsilon_{zx}=-r/3,\quad \epsilon_{xy}=-2r/3. \tag{35}$$

Although Fig. 4 shows that (11̄1) [011] and (1̄11) [101] [Nos. (9) and (11)] are the most likely systems to operate according to the Tucker criterion, they are not enough to satisfy the strain equations (35). Additional slip on the cross-slip systems (111̄) [101] and (111̄) [011] [Nos. (4) and (5)] may then be assumed. Thus

$$\begin{aligned} 2\epsilon_{xx}&=S_4-S_{11},\\ 2\epsilon_{yy}&=S_5-S_9,\\ 2\epsilon_{zz}&=-S_4-S_5+S_9+S_{11},\\ 4\epsilon_{yz}&=S_4+S_{11},\\ 4\epsilon_{zx}&=S_5+S_9,\\ 4\epsilon_{xy}&=S_4+S_5+S_9+S_{11}. \end{aligned} \tag{36}$$

Solution of Eqs. (35) and (36) gives

$$S_4=S_5=-(r/3),\quad S_9=S_{11}=r. \tag{37}$$

Hence

$$E_{\mathrm{LF}}=(1/48)K_{\mathrm{LF}}r(3\alpha_3^2-7\alpha_2\alpha_3-7\alpha_3\alpha_1). \tag{38}$$

On the rolling (112) plane, E_{LF} is minimum along the [1̄10] rolling direction. Likewise,

$$E_{\mathrm{SC}}=-(1/72)K_{\mathrm{SC}}r(\alpha_2\alpha_3+\alpha_3\alpha_1+2\alpha_1\alpha_2). \tag{39}$$

TABLE II. Summary of calculated anisotropy energies based on slip-induced directional order theory.

A. Wire drawing WA	E_{LF}	Easy axis	E_{SC}	Easy axis
1. ⟨001⟩	$(K_{LF}r/16)\alpha_3^2$	⊥WA	0	⋯
2. ⟨111⟩	$-(K_{LF}r/8)(\alpha_1\alpha_2+\alpha_2\alpha_3+\alpha_3\alpha_1)$	WA	$-(K_{SC}r/48)(\alpha_1\alpha_2+\alpha_2\alpha_3+\alpha_3\alpha_1)$	WA

B. Rolling RP RD	E_{LF}	Easy axis on RP	E_{SC}	Easy axis on RP
1. (001)[100][a]	a. $-(K_{LF}r/8)\alpha_2^2$	TD	a. 0	⋯
	b. $-(K_{LF}r/8)\alpha_2^2$	TD	b. $(K_{SC}r/24)\alpha_2\alpha_3$	~TD
2. (001)[110][a]	a. $(K_{LF}r/16)\alpha_3^2$	⋯	a. $(K_{SC}r/24)\alpha_1\alpha_2$	TD
	b. $-(K_{LF}r^2/4)\cos^2\theta$	RD	b. $[K_{SC}r(1-2r)/24]\cos^2\theta$	TD
3. (110)[001][a]	a. $(K_{LF}r/16)\alpha_3^2$	TD	a. $(K_{SC}r/24)\alpha_1\alpha_2$	TD
	b. $(K_{LF}r/16)\alpha_3^2$	TD	b. $(K_{SC}r/24)\alpha_1\alpha_2$	TD
4. (110)[$\bar{1}$12][b]	a. $(K_{LF}r/24)[(\alpha_1+\alpha_3)^2+(\alpha_2-\alpha_3)^2]$	[$\bar{1}$11], 20° from RD	a. $(K_{SC}r/36)\alpha_1\alpha_2$	[$\bar{1}$10], 55° from RD
	b. $(5K_{LF}r/48)[(\alpha_1+\alpha_3)^2+(\alpha_2-\alpha_3)^2]$	[$\bar{1}$11], 20° from RD	b. $(K_{SC}r/72)(2\alpha_1\alpha_2-\alpha_2\alpha_3+\alpha_3\alpha_1)$	near [$\bar{1}$11], 25° from RD
5. (110)[$\bar{1}$10]	$(K_{LF}r/8)\alpha_3^2$	RD	0	⋯
6. (111)[$\bar{1}\bar{1}$2]	$\sim(7K_{LF}r/48)(\alpha_1-\alpha_2)^2$	RD	$-(K_{SC}r/144)(2\alpha_2\alpha_3+2\alpha_3\alpha_1+5\alpha_1\alpha_2)$	RD
7. (112)[$\bar{1}$10][b]	a. $(K_{LF}r/24)(\alpha_3^2-2\alpha_2\alpha_3-2\alpha_3\alpha_1)$	RD	a. $-(K_{SC}r/36)\alpha_1\alpha_2$	TD
	b. $(K_{LF}r/48)(3\alpha_3^2-7\alpha_2\alpha_3-7\alpha_3\alpha_1)$	RD	b. $-(K_{SC}r/72)(\alpha_2\alpha_3+\alpha_3\alpha_1+2\alpha_1\alpha_2)$	TD
8. (112)[$\bar{1}\bar{1}$1]	$(K_{LF}r/48)[(\alpha_1+\alpha_3)^2+(\alpha_2+\alpha_3)^2+7(\alpha_1-\alpha_2)^2]$	RD	$(K_{SC}r/144)(2\alpha_2\alpha_3+2\alpha_3\alpha_1-5\alpha_1\alpha_2)$	RD

[a] These three cases have been analyzed and studied by CSI.[5] a, calculation based on homogeneous slip; b, based on observed slip.
[b] a, calculation based on slip on systems of maximum effective Schmid factor; b, based on additional slip systems to achieve strain compatibility. WA wire axis; RP, rolling plane; RD, rolling direction; TD, transverse direction.

On the (112) plane, E_{SC} is maximum along [$\bar{1}$10]. Hence this anisotropy opposes that obtained from L.F. deformation. If only systems (9) and (11) operate, despite strain incompatibility,

$$E_{LF}=(1/24)K_{LF}r(\alpha_3^2-2\alpha_2\alpha_3-2\alpha_3\alpha_1) \qquad (40)$$

and

$$E_{SC}=-(1/36)K_{SC}r\alpha_1\alpha_2. \qquad (41)$$

The predicted easy directions on (112) are thus the same as the four slip system case.

(*e*) (*112*) [$\bar{1}\bar{1}1$] *Rolling*

The specimen coordinate axes are now x'–[112], y'–[$\bar{1}$10], and z'–[$\bar{1}\bar{1}$1], Fig. 4. The transformation matrix is

	x'	y'	z'
x	$\frac{1}{\sqrt{6}}$	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{3}}$
y	$\frac{1}{\sqrt{6}}$	$\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{3}}$
z	$\frac{2}{\sqrt{6}}$	0	$\frac{1}{\sqrt{3}}$

and

$$\epsilon_{xx}=r/6, \quad \epsilon_{yy}=r/6, \quad \epsilon_{zz}=-r/3,$$
$$\epsilon_{yx}=-2r/3, \quad \epsilon_{zx}=-2r/3, \quad \epsilon_{xy}=r/6. \qquad (42)$$

The operating slip systems are most likely (111) [0$\bar{1}$1], (111) [$\bar{1}$01], (1$\bar{1}$1) [110], and ($\bar{1}$11) [110], or Nos. (1), (2), (7), and (12). Then

$$\begin{aligned} 2\epsilon_{xx}&=-S_2+S_7-S_{12},\\ 2\epsilon_{yy}&=-S_1-S_7+S_{12},\\ 2\epsilon_{zz}&=S_1+S_2,\\ 4\epsilon_{yz}&=S_2+S_7+S_{12},\\ 4\epsilon_{zx}&=S_1+S_7+S_{12},\\ 4\epsilon_{xy}&=-S_1-S_2. \end{aligned} \qquad (43)$$

Solution of Eqs. (42) and (43) gives

$$S_1=S_2=-(r/3), \quad S_7=S_{12}=-(7/6)r. \qquad (44)$$

Hence

$$E_{LF}=(1/48)K_{LF}r[(\alpha_1+\alpha_3)^2+(\alpha_2+\alpha_3)^2+7(\alpha_1-\alpha_2)^2]. \qquad (45)$$

Equation (45) tells us that on the (112) rolling plane the [$\bar{1}\bar{1}$1] direction (rolling direction) is the easy axis. Similarly,

$$E_{SC}=(1/144)K_{SC}r(2\alpha_2\alpha_3+2\alpha_3\alpha_1-5\alpha_1\alpha_2), \qquad (46)$$

which again places the easy direction at [$\bar{1}\bar{1}$1] as far as the (112) rolling plane is concerned.

DISCUSSION

The results of the preceding calculations are summarized in Table II, together with the three cases studied by Chikazumi *et al.*[5] The positions of the induced easy axis on the rolling plane are indicated. These positions are perhaps the most significant predictions of the theory, for they can be checked conveniently by magnetic torque measurements on a disk